# The cubic fourth order Schrödinger equation on a star graph

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**Abstract**

In this talk, we present some recent results related to the wellposedness of the cubir fourth order Schrodinger equation on star graph structure $\mathcal{G}$, [9]:

$$i\partial_t u - \partial_x^4 u - \lambda |u|^2 u = 0, \tag{1}$$

Here, we consider $\mathcal{G}$ composed by $N$ edges parameterized by half-lines $(0, +\infty)$ attached with a common vertex $\nu$. With this structure the manuscript proposes to study the well-posedness of a dispersive model on star graphs with three appropriated vertex conditions by using the *boundary forcing operator approach*, more precisely, we give positive answer for the Cauchy problem in low regularity Sobolev spaces. We have noted that this approach seems very efficient, since this allows to use the tools of Harmonic Analysis, for instance, the Fourier restriction method of Bourgain, while for the other known standard methods to solve partial differential partial equations on star graphs are more complicated to capture the dispersive smoothing effect in low regularity. The arguments presented in this work has prospects to be applied for others nonlinear dispersive equations on the context of star graphs with unbounded edges.

**Work in collaboration with R.A. Capistrano-FilhoPazoto (UFPE, Brazil) and M. Cavalcante (UFAL, Brazil)**

## References

[1] R. Adami, C. Cacciapuoti, D. Finco and D. Noja, *Variational properties and orbital stability of standing waves for NLS equation on a star graph,* J. Differential Equations, 257, 3738–3777 (2014).

[2] R. Adami, C. Cacciapuoti, D. Finco and D. Noja, *Stable standing waves for a NLS on star graphs as local minimizers of the constrained energy,* J. Differential Equations, 260, 7397–7415 (2016).

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[3] J. Angulo and N. Goloshchapova, *On the orbital instability of excited states for the NLS equation with the $\delta$-interaction on a star graph,* Discrete Contin. Dyn. Syst. A., 38, no. 10, 5039–5066 (2018).

[4] J. Angulo and N. Goloshchapova, *Extension theory approach in the stability of the standing waves for the NLS equation with point interactions on a star graph*, Adv. Differential Equations 23, no. 11-12, 793–846 (2018).

[5] K. Ammari and E. Crépeau, *Feedback Stabilization and Boundary Controllability of the Korteweg–de Vries Equation on a Star-Shaped Network*, SIAM J. Control Optim., 56, (3), 1620–1639 (2018).

[6] M. Ben-Artzi, H. Koch and J.-C Saut, *Dispersion estimates for fourth order Schrödinger equations*, C. R. Acad. Sci. Paris Sér. I Math, 330, 87–92 (2000).

[7] G. Berkolaiko and P. Kuchment, *Introduction to Quantum Graphs*, Mathematical Surveys and Monographs, 186, Amer. Math. Soc., Providence, RI, (2013).

[8] R. A. Capistrano-Filho and M. Cavalcante, *Stabilization and control for the biharmonic Schrödinger equation*, Preprint arXiv:1807.05264 (2018).

[9] R. A. Capistrano-Filho, M. Cavalcante and F. A. Gallego, *Forcing operator on star graph applied in the cubic fourth order Schrödinger equation.* arXiv preprint arXiv:1909.06812 (2019).

[10] R. A. Capistrano-Filho, M. Cavalcante and F. A. Gallego, *Lower regularity solutions of the biharmonic SchrÃ¶dinger equation in a quarter plane*, Preprint arXiv:arXiv:1812.11079 (2018).

[11] M. Cavalcante, *The initial-boundary-value problem for some quadratic nonlinear Schrödinger equations on the half-line*, Differential and Integral Equations, Vol. 30, no. 7/8, 521–554 (2017).

[12] M. Cavalcante, *The Korteweg–de Vries equation on a metric star graph*, Z. Angew. Math. Phys. 69: 124 (2018).

[13] M. Cavalcante and A. J. Corcho, *The initial boundary value problem for the Schrödinger–Korteweg–de Vries system on the half-line*, Communications in Contemporary Mathematics, v. 2019, 1–47 (2019).

[14] E. Cerpa, E. Crépeau and C. Moreno, *On the boundary controllability of the Korteweg-de Vries equation on a star-shaped network*, IMA Journal of Mathematical Control and Information, https://doi.org/10.1093/imamci/dny047 (2019).

[15] J. Colliander and C. Kenig, *The generalized Korteweg-de Vries equation on the half-line*, Comm. Partial Differential Equations, Vol.27 2187–2266 (2002).

[16] S. Cui and C. Guo, *Well-posedness of higher-order nonlinear Schrödinger equations in Sobolev spaces $H^s(R^n)$ and applications*, Nonlinear Analysis, 67:687–707 (2007).

[17] F. Gregorio and D. Mugnolo, *Bi-Laplacians on graphs and networks*, J. Evol. Equ. (2019). https://doi.org/10.1007/s00028-019-00523-7

[18] J. Holmer, *The initial-boundary value problem for the 1d nonlinear Schrödinger equation on the half-line*, Differential Integral Equations 18, 6470–668 (2005).

[19] J. Holmer, *The initial-boundary value problem for the Korteweg-de Vries equation*, Comm. in Partial Differential Equations, Vol.31, 1151–1190 (2006).

[20] L. I. Ignat, A. F. Pazoto and L. Rosier, *Inverse problem for the heat equation and the Schrödinger equation on a tree*, Inverse Problems, 28(1), (2011).

[21] P. Kuchment, *Quantum graphs, I. Some basic structures,* Waves Random Media, 14, 107–128 (2004).

[22] D. Mugnolo, D. Noja and C. Seifert, *Airy-type evolution equations on star graphs,* Analysis & PDE. 11 (7), 1625–1652 (2018).

[23] B. Pausader, *The cubic fourth-order Schrödinger equation*, J. Funct. Anal., 256, 2473–2517 (2009).

[24] T. Tsutsumi, *Strichartz estimates for Schrödinger equation of fourth order with periodic boundary condition*, Kyoto university, 11pp, (2014).